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A Comparison of MOEA/D, NSGA II and SPEA2 Algorithms

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ABSTRACT

The optimization of multi-objective problems is currently an important area of research and development. The importance gained by this type of problem has given rise to the development of multi-objective metaheuristics to attain solutions for such problems. In this paper, an experimental comparison of MOEA/D (Multiobjective Evolutionary Algorithm based on Decomposition), NSGA II (Nondominated Sorting Genetic Algorithm II), and SPEA2 (Strength Pareto Evolutionary Algorithm 2) using ZDT benchmark, has been done to determine which multi-objective metaheuristic has the best performance with respect to a problem. The results thus obtained are compared and analyzed based on three performance metrics namely Hyper volume, GD, and IGD that evaluate the dispersion of the solutions and its proximity to it.

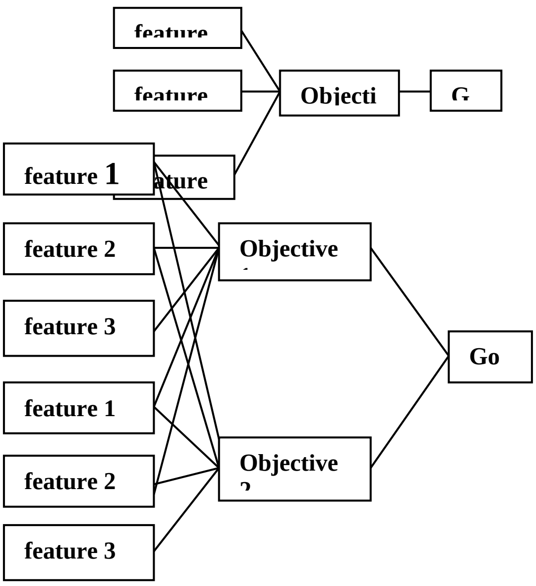
***Keywords:* —** Multiobjective optimization; Multiobjective evolutionary algorithm based on Decomposition, Nondominated sorting genetic algorithm-II; Strength pareto evolutionary algorithm2.

I. INTRODUCTION

Many real-world decision making problems need to achieve several objectives such as minimized risks, maximized reliability, minimized deviations from desired levels, and minimized cost etc. The main goal of single-objective (SO) optimization is to find the "best" solution, which corresponds to the minimum or maximum value of a single objective function that lumps all different objectives into one. This type of optimization is useful as a tool that provides decision makers the insights into the nature of the problem, but usually lacks to provide a set of alternative solutions that have trade-off among different objectives against each other. On the contrary, in a multi-objective optimization with conflicting objectives, there is no single optimal solution (Rajani et al 2017). The interaction among different objectives gives rise to a set of compromised solutions, largely known as the trade-off, non-dominated, non-inferior or Pareto-optimal solutions (Savic, 2002).

Optimization is used to find out one or more feasible solutions that give(s) the best value(s) for one or more objectives of some function. When only one objective function is involved in the optimization problem, the task of finding the best possible solution is called single objective optimization as shown in figure 1(a). On the other hand, when more than one objective function are involved in the optimization problem, the task of finding one or more best possible solution(s) is known as multi-objective optimization as shown in figure 1(b) (Bandyopadhyay and Saha, 2013).

Figure 1 (a) Single objective problem formulation and **(b)** Multi-objective problem formulation



The task of simultaneously optimizing more than one conflicting objectives with respect to a set of certain constraints is dealt with the help of multi-objective optimization. In multi-objective optimization problem, if optimization of one objective automatically leads to optimization of another objective, then this kind of problem is not considered as a multi-objective optimization problem. However, in many real-life situations we come across problems where an attempt to improve one objective leads to degradation of the other objective. Such problems belong to the class of multi-objective optimization problems (Deb, 2014).

Mathematically, the multi-objective problem could be written as follows:

Maximize/minimize

$$p_a(y) \qquad a = 1,2,\dots,N \qquad (1)$$

Subject to constraints:

$$q_b(y) \geq 0 \qquad b = 1,2,\dots,J \qquad (2)$$

$$r_c(y) = 0 \qquad c = 1,2,\dots,K \qquad (3)$$

$$y_d^{(l)} \leq y_d \leq y_d^{(u)} \qquad d = 1,2,\dots,M \qquad (4)$$

where parameter y is an n dimensional vector with n design or decision variables, $y = (y_1, y_2, \dots, y_n)$. The constraints in equation (4) are called variables bound, that restrict each decision variable y_d to take a value within a lower $y_d^{(l)}$ and an upper $y_d^{(u)}$ bound. These bounds make up a decision space, D. A solution y that satisfies all the $(J+K)$ constraints and all the $2M$ variable bounds is called a *feasible solution* otherwise it is known as an *infeasible solution*. The set of all feasible solutions is called the *feasible region* or *search space*, S (Deb, 2014).

Given two vectors $y, y' \in \Omega$ (where Ω is the decision space), the vector x is said to dominate y', $y > y'$, iff y is not worse than y' in any objective function and it is strictly better in at least one objective function (Deb, 2014). If neither y dominates y', nor y' dominates y, y and y' are said to be no-comparable, denoted as $y \sim y'$.

For a given MOP, the Pareto optimal set $(P*)$ is the set containing all the solutions that are non-dominated with respect to Ω. It can be denoted as (Veldhuizen and Veldhuizen, 1999):

$$P* = \{y \in \Omega | \neg \exists y' \in \Omega \; such \; that \; y' > y\}$$

Then, for a given MOP and its corresponding Pareto optimal set $(P*)$, the Pareto optimal front $(PF*)$ is the result of mapping $P*$ to Λ (where Λ is the objective space). $PF*$ is defined as (Veldhuizen and Veldhuizen, 1999):

$$PF* = \{F(y) \in \Lambda | y \in P*\}$$

where $F(y)$ is the vector function containing K objective function.

An approximation set is defined by Zitzler et al. as follows (Zitzler et al., 2003): let $A \subseteq \Lambda$ be a set of objective vectors. A is called an approximation set if any element of A does not dominate or is not equal to any other objective vector in A. The set of all approximation sets is denoted as Z.

The rest of the paper is organized as follows: Section 2 introduces brief review of multi-objective optimization algorithms. Section 3 talks about the basics of three multi-objective optimization techniques i.e. MOEA/D, NSGA II, and SPEA2. Section 4 covers experimentation, results and discussion. Section 5 concludes the work.

II. REVIEW OF ALGORITHMIC CONCEPTS OF MOEA/D, NSGA II, AND SPEA2

Optimimizing multiple objectives in a problem is currently an important area of research and development. Recently, researchers focused on development of multiple metaheuristics for solving multi-objective problems. The following section gives the review of algorithmic concepts of MOEA/D, NSGA II, and SPEA2.

2.1 MOEA/D (Multiobjective Evolutionary Algorithm based on Decomposition)

MOEA/D (Zhang and Li 2007) explicitly decomposes the MOP into scalar optimization sub problems. It solves these sub problems simultaneously by evolving a population of solutions. At each generation, the population is composed of the best solution found so far (i.e. since the start of the run of the algorithm) for each subproblem. The neighborhood relations among these subproblems are defined based on the distances between their aggregation coefficient vectors. The optimal solutions to two neighboring subproblems should be very similar. Each subproblem (i.e., scalar aggregation function) is optimized in MOEA/D by using information only from its neighboring subproblems.

2.2 NSGA-II (Nondominated Sorting Genetic Algorithm II)

The multi-objective evolutionary algorithms (MOEA) suffered lots of drawbacks such as: 1. Computational complexity $(O(MN^3))$ where M is the number of objective and N is the population size to $(O(MN^2))$. 2. Non-elitism approach and 3. Need for specifying a sharing parameter. To overcome these difficulties, K. Deb et al. (2002) proposed an elitist non-dominated Sorting GA (NSGA-II). In NSGA-II, the parent population P_p is used to create the offspring (child) population P_0. To ensure a better spread among the solution, a niching strategy is used to choose the diverse set of solution from the set (i.e. parent population + offspring population) followed by crowded tournament selection, crossover and mutation operator (Rajani et al 2017).

2.3 SPEA2 (Strength Pareto Evolutionary Algorithm 2)

SPEA2 (Zitzler, Laumanns, and Thiele 2001) is an extension of SPEA (Strength Pareto Evolutionary Algorithm) (Zitzler and Thiele 1999) that resolves the following weakness of SPEA i.e. 1) Fitness assignment, 2) Density estimation, 3) Archive truncation. SPEA uses a regular population and an archive (external set). Starting with an initial population and an empty archive the following steps are performed per iteration. First, all nondominated population members are copied to the archive; any dominated individuals or duplicates (regarding the objective values) are removed from the archive during this update operation. If the size of the updated archive exceeds a predefined limit, further archive members are deleted by a clustering technique which preserves the characteristics of the nondominated front. Afterwards, fitness values are assigned to both archive and population members.

III. EXPERIMENTATION and Result Discussion

3.1 Benchmark Functions Used

The definition of the benchmark functions used for experimentation is as follows (Table 1):

Table 1 Constraint Benchmark Functions

$f_1(x) = x_1$ $f_2(x) = g(x)[1 - \sqrt{\frac{x_1}{g(x)}}]$ $g(x)$ $g(x) = 1 + 9(\sum_{i=2}^{n} x_i)/(n-1)$
$f_1(x) = x_1$ $f_2(x) = g(x)[1 - (x_1/g(x))^2]$ $g(x)$ $g(x) = 1 + 9(\sum_{i=2}^{n} x_i)/(n-1)$
$f_1(x) = x_1$ $f_2(x) = g(x)[1 - \sqrt{x_1/g(x)} - x_1/g(x)\sin(10\pi x_1)]$ $g(x)$ $g(x) = 1 + 9(\sum_{i=2}^{n} x_i)/(n-1)$
$f_1(x) = x_1$ $f_2(x) = g(x)[1 - \sqrt{x_1/g(x)}]$ $g(x)$ $g(x) = 1 + 10(n-1) + \sum_{i=2}^{n}[x_i^2 - 10\cos(4\pi x_i)]$
$f_1(x) = 1 - e^{(-4x_1)} sin^6(6\pi x_1)$ $f_2(x) = g(x)\left[1 - (\frac{f_1(x)}{g(x)})^2\right]$ $g(x)$ $g(x) = 1 + 9\left[\frac{\sum_{i=2}^{n} x_i}{(n-1)}\right]^{\frac{1}{4}}$

3.2 Parameter Setting for Involved Algorithms

The following table indicates the parameter settings for the three algorithms.

Table 2 Parameter Settings for NSGA II, MOEA/D, and SPEA2

	Population	Mutation	Crossover
	80	0.6	0.9
NSGA II	100	0.6	0.9
	120	0.6	0.9
	80	0.5	-
MOEA/D	100	0.5	-
	120	0.5	-
	80	0.9	-
SPEA2	100	0.9	-
	120	0.9	-

3.3 Performance Metrics

For the performance measure, as many as three different performance metrics namely, Hypervolume, Generational Distance (GD), and Inverted Generational Distance (IGD) have been used (Rajani et al 2017). These are defined as:

The Hypervolume (HV) (Auger et al, 2009), also known as S metric, hyper-area or Lebesgue measure, is a unary metric that measures the size of the objective space covered by an approximation set. HV considers all three aspects: accuracy, diversity and cardinality, being the only unary metric with this capability. An algorithm with a large value of HV is desirable. Hypervolume is calculated as:

$$HV = volume\left(\cup_{i=1}^{|A|} v_i\right)$$

Generational Distance (GD) (Riquelme et al, 2015) takes as reference an approximation set A and calculates how far it is from the Pareto optimal front $PF*$ (or reference set R). This unary measure considers the average Euclidean distance between the members of A and the nearest member of $PF*$. It can be noticed that GD considers only one aspect of A: the accuracy. An algorithm having a small value of GD is better. It is calculated as:

$$GD = \frac{\left(\sum_{i=1}^{|A|} d_i^p\right)^{1/p}}{|A|}$$

Inverted Generational Distance (IGD) (Riquelme et al, 2015) is an inverted variation of GD but it is significantly different from GD: i) it calculates the minimum Euclidean distance (instead of the average distance) between an approximation set A and the Pareto optimal front $PF*$, ii) IGD uses as reference the solutions in $PF*$ (and not the solutions in A) to calculate the distance between the two sets and iii) if sufficient members of $PF*$ are known, IGD could measure both the diversity and the convergence of A. It is calculated as:

$$IGD = \frac{\sqrt{\sum_{i=1}^{n} d_i^2}}{n}$$

where n is the number of true pareto optimal solutions and d_i indicates the Euclidean distance between the ith true pareto optimal solution and the closest obtained pareto optimal solutions in the reference set.

3.4 Results and Discussion

The above performance metrics allow us to quantitatively compare MOEA/D, NSGA II, and SPEA2. All the algorithms are run 20 times on the test problems and the statistics results of these 20 runs are provided in Tables 3-11.

The comparison of the results over five multi-objective test functions i.e. ZDT1, ZDT2, ZDT3, ZDT4 and ZDT6 for three algorithms namely MOEA/D, NSGA II, and SPEA2 is given in table below:

Population 80:

Table 3 provides statistical results of the algorithms for Hypervolume (HV) on population 80. Since HV is the metric that considers accuracy and diversity of an algorithm. This table shows that MOEA/D algorithm outperforms others on statistical metric mean for ZDT1, ZDT2, ZDT4 and ZDT6. For ZDT3, NSGA II outperforms MOEA/D and SPEA 2 in terms of mean. MOEA/D also outperforms others in terms of statistical metric standard deviation.

Table 3 Statistical results for Hypervolume on Population 80 on ZDT benchmark

	MOEA/D		**NSGA II**		**SPEA2**	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
ZDT1	6.61E-01	1.30E-04	6.59E-01	3.19E-03	6.60 E-01	3.45E-04
ZDT2	3.27 E-01	1.02 E-04	3.26E-01	2.46E-03	3.26 E-01	2.39E-03
ZDT3	5.13 E-01	5.30E-05	5.15E-01	1.80E-04	5.14 E-01	3.20E-04
ZDT4	6.60 E-01	2.57 E-04	6.59E-01	3.52E-03	6.49 E-01	1.52E-02
ZDT6	4.00 E-01	5.76E-08	3.98E-01	1.00E-03	3.81 E-01	2.22E-03

Table 4 Statistical results for GD on Population 80 on ZDT benchmark

	MOEA/D		**NSGA II**		**SPEA2**	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
ZDT1	1.15 E-04	1.35E-05	8.44 E-04	1.07E-04	2.11E-04	2.52E-05
ZDT2	5.90 E-05	2.42E-06	8.24 E-04	6.00E-05	1.53E-04	3.53E-05
ZDT3	1.00 E-04	2.75E-06	5.62 E-04	4.12E-05	1.37E-04	1.77E-05
ZDT4	1.02 E-04	1.22E-05	8.61 E-04	8.96E-05	5.76E-04	3.03E-04
ZDT6	4.99 E-05	3.96E-07	7.84 E-04	4.96E-05	1.46E-03	1.82E-04

Table 5 Statistical results for IGD on Population 80 on ZDT benchmark

	MOEA/D		NSGA II		SPEA2	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
ZDT1	1.99E-04	6.37E-07	8.47E-04	8.80E-05	1.49E-04	3.81E-06
ZDT2	1.76E-04	4.95E-08	8.20E-04	5.73E-05	2.21E-04	3.07E-04
ZDT3	3.37E-04	1.72E-06	5.64E-04	3.30E-05	1.23E-04	6.04E-06
ZDT4	2.06E-04	1.28E-06	8.61E-04	7.52E-05	9.36E-04	1.64E-03
ZDT6	1.75E-04	5.76E-09	7.79E-04	5.76E-05	4.70E-04	4.70E-05

The statistical results of the algorithms on ZDT benchmark for GD are shown in Table 4. It has been observed from the table that for statistical metric mean, NSGA II has the better results than others except on ZDT4. For ZDT4, SPEA2 has maximum value for mean. GD is the performance metric that shows the accuracy of an algorithm. So it can be stated that NSGA II algorithm is superior except for ZDT4. For statistical metric standard deviation, MOEA/D outperform better than others.

Table 5 shows the statistical results of the algorithms on ZDT benchmark functions for IGD. It has been observed from the table that NSGA II has better results for ZDT1, ZDT2, ZDT3 and ZDT6. For ZDT4, SPEA2 has good results for mean. It has also been observed that for statistical metric standard deviation, MOEA/D again outperform better than others.

Population 100:

The statistical results of MOEA/D, NSGA II, and SPEA2 on ZDT benchmark on population 100 for GD, IGD and Hypervolume are shown in Tables 6-8.

The statistical results of NSGA II, Abyss and OMOPSO on ZDT benchmark on population 100 for hypervolume metric is shown in Table 6. According to this, MOEA/D outperforms others for statistical metrics mean except for ZDT3. For ZDT3, NSGA II has greater value for mean. For statistical metric standard deviation, MOEA/D dominates others. It demonstrates the supremacy of MOEA/D algorithm.

The statistical results of MOEA/D, NSGA II, and SPEA2 on ZDT benchmark on population 100 for GD metric is shown in Table 7. According to this, NSGA II outperforms others on ZDT1, ZDT2, ZDT3, and ZDT4. For ZDT6, SPEA2 outperforms others. Again for statistical metric standard deviation, MOEA/D has best values.

Table 6 Statistical results for Hypervolume on Population 100 on ZDT benchmark

	MOEA/D		NSGA II		SPEA2	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
ZDT1	6.61E-01	1.11E-04	6.60E-01	1.34E-03	6.60E-01	3.30E-04
ZDT2	3.28E-01	1.09E-04	3.27E-01	1.45E-03	3.26E-01	5.81E-04
ZDT3	5.14E-01	4.41E-05	5.15E-01	1.03E-03	5.14E-01	2.18E-04
ZDT4	6.61E-01	9.76E-05	6.60E-01	5.53E-03	6.50E-01	8.61E-03
ZDT6	4.01E-01	5.40E-08	3.99E-01	2.14E-03	3.79E-01	2.28E-03

Table 7 Statistical results for GD on Population 100 on ZDT benchmark

	MOEA/D		NSGA II		SPEA2	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
ZDT1	8.89E-05	1.15E-05	6.19E-04	4.60E-05	2.24E-04	2.07E-05
ZDT2	5.08E-05	4.79E-06	5.92E-04	3.15E-05	1.89E-04	4.36E-05
ZDT3	9.17E-05	2.72E-06	4.22E-04	2.66E-05	1.42E-04	1.32E-05
ZDT4	7.25E-05	1.15E-05	6.88E-04	1.03E-04	6.41E-04	3.97E-04
ZDT6	4.16E-05	3.46E-07	5.89E-04	3.79E-05	1.61E-03	1.68E-04

Table 8 Statistical results for IGD on Population 100 on ZDT benchmark

	MOEA	NSGA II	SPEA2

	MOEA/D		NSGA II		SPEA2	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
ZDT1	1.60E-04	6.69E-07	6.19E-04	5.56E-05	1.53E-04	4.60E-06
ZDT2	1.41E-04	1.60E-07	5.91E-04	3.16E-05	1.57E-04	7.64E-06
ZDT3	2.83E-04	1.67E-06	4.18E-04	2.09E-05	1.25E-04	6.29E-06
ZDT4	1.68E-04	2.19E-06	6.86E-04	1.07E-04	9.23E-04	9.81E-04
ZDT6	1.40E-04	4.58E-09	5.83E-04	4.41E-05	5.15E-04	5.00E-05

The statistical results of MOEA/D, NSGA II, and SPEA2 on ZDT benchmark on population 100 for IGD metric is shown in Table 8. According to this, again NSGA II outperforms others except ZDT4 for mean. For ZDT4, SPEA2 has maximum value for mean. Again for statistical metric standard deviation, MOEA/D dominates others.

Population 120:

The statistical results of MOEA/D, NSGA II, and SPEA2 on ZDT benchmark on population 120 for GD, IGD and Hypervolume metric are shown in table 9-11.

The statistical results of MOEA/D, NSGA II, and SPEA2 on ZDT benchmark on population 120 for Hypervolume metric is shown in Table 9. According to this, MOEA/D outperforms others in terms of mean and standard deviation except ZDT3. For ZDT3, NSGA II has the maximum value for mean.

Table 10 shows the statistical results of MOEA/D, NSGA II, and SPEA2 on ZDT benchmark on population 120 for GD metric. According to this, NSGA II outperforms others on ZDT1, ZDT2, ZDT3, and ZDT6. For ZDT4, SPEA2 outperforms others. For standard deviation, MOEA/D has best values.

Table 9 Statistical results for Hypervolume on Population 120 on ZDT benchmark

	MOEA/D		NSGA II		SPEA2	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
ZDT1	6.62E-01	1.75E-04	6.61E-01	1.20E-03	6.60E-01	2.84E-04
ZDT2	3.29E-01	8.51E-05	3.28E-01	4.54E-04	3.23E-01	1.16E-02
ZDT3	5.14E-01	3.64E-05	5.16E-01	3.19E-04	5.14E-01	3.66E-04
ZDT4	6.62E-01	9.90E-05	6.61E-01	5.18E-03	6.52E-01	6.41E-03
ZDT6	4.02E-01	2.30E-07	4.00E-01	3.58E-03	3.74E-01	2.65E-03

Table 10 Statistical results for GD on Population 120 on ZDT benchmark

	MOEA/D		NSGA II		SPEA2	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
ZDT1	8.03E-05	1.16E-05	4.52E-04	1.61E-05	2.43E-04	2.45E-05
ZDT2	5.15E-05	2.90E-06	4.55E-04	2.07E-05	2.21E-04	6.20E-05
ZDT3	8.60E-05	2.79E-06	3.24E-04	2.18E-05	1.46E-04	1.67E-05
ZDT4	6.74E-05	9.57E-06	5.81E-04	1.67E-04	5.89E-04	3.40E-04
ZDT6	3.81E-05	1.51E-07	5.04E-05	5.04E-05	2.02E-03	1.96E-04

Table 11 Statistical results for IGD on Population 120 on ZDT benchmark

	MOEA/D		NSGA II		SPEA2	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
ZDT1	1.34E-04	8.98E-07	4.57E-04	2.37E-05	1.55E-04	2.76E-06
ZDT2	1.17E-04	1.54E-07	4.55E-04	2.82E-05	3.89E-04	1.02E-03
ZDT3	2.30E-04	1.11E-06	3.26E-04	1.73E-05	1.24E-04	7.31E-06
ZDT4	1.40E-04	2.13E-06	5.80E-04	1.66E-04	6.20E-04	7.36E-04
ZDT6	1.16E-04	6.89E-09	5.05E-04	5.32E-05	6.36E-04	5.89E-05

Table 11 shows the statistical results of MOEA/D, NSGA II, and SPEA2 on ZDT benchmark on population 120 for IGD metric. According to this, NSGA II outperforms others on ZDT1, ZDT2 and ZDT3. For ZDT4 and ZDT6, SPEA2 outperforms others. Again for standard deviation, MOEA/D has best values.

IV. CONCLUSIONS

In this paper, we have performed an experimental comparison among three algorithms for multi-objective optimization. To evaluate the performance of the algorithms we have used five well-known benchmark problems (ZDT) by three unary metrics i.e. Hypervolume, GD, IGD. According to experiments with test functions examined and the parameter settings used, we can conclude that for Hypervolume metric MOEA/D yields the best performance in terms of mean on population 80,100 and 120. For GD and IGD metric, MOEA/D yields best results for standard deviation and NSGA II yields best results for mean on population 80,100 and 120.

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