

Fibonacci Coding for Lossless Data Compression – A Review

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ABSTRACT

In this paper, Fibonacci coding data compression technique reviewed. Initially, Fibonacci encoding performed for the input with fewer amounts of symbols. Then decoding for the obtained result achieved. It regenerates the original uncompressed data. Finally, input with more number of symbols taken for compression. The average bits, compression ratio, and, space savings also calculated.

Keywords:- Fibonacci Coding, Compression, Encoding, Decoding.

I. INTRODUCTION

Data compression defined as the representation of data in such a way that, the storage area needed for target data is less than that of, the size of the input data [1]. The data compression used in plenty of data processing areas. In ASCII code, we have 256 characters represented by the different numbers. Ex. 'a' represented, numerically as 97. The frequency of each ASCII code differs from each other. If text data used as an input, some characters occur most frequently, and many characters never utilized in the input. The alphabet, digits and some special characters used mostly. In the alphabet, the most used characters are vowels. The 256 symbols never used frequently.

The variations in this frequency of characters need data compression. The fixed length code, need to be replaced by variable length code. There exist many research papers on variable length code for data compression [2, 3, 4, and 5]. Fibonacci coding generates variable length codes. It uses traditional methods of replacing input characters by specific code like code words. It uses Fibonacci series to implement the compression.

II. RELATED WORK

Apostolico and Fraenkel [1987] generated variable length code for data compression [6]. They used the concept invented by Leonardo of Pisa [1202] known as Fibonacci for mathematical calculation [7]. It translated by sigler [2002] as Fibonacci's Liber Abaci [8].

III. FIBONACCI CODING

The Fibonacci series concept developed from the calculation of rabbit reproduction [7]. The rabbit pairs, ready for reproduction at one month. Then next month it produces another pair. Therefore, the rabbit pair produces another rabbit pair after the second month. Then each month it produces a pair. If all pair survives and performs reproduction as usual then at the end of the first month (i.e. starting with the second month), we have one pair. After second, third, and fourth month, we will have one, two, three, and five pairs. In the Table I, the explanation for Fibonacci series through rabbit reproduction had given.

The numbers in the red color (entire first column) represents the month. The numbers in the blue color (whole first row) represents the number given for the rabbit pair. The numbers in the green color (whole last column) represents the total number of the pair at the starting of the month. The reproduction process represented by the brown colored number. The brown colored zero shows that the rabbit pair is ready for the reproduction. The brown colored one shows that the rabbit pair reproduced a rabbit pair (Except second row and second column one). The first pair represented in the table as the value 1 in the second row and second column. The second pair, third pair represented in the table as the value 1 in the third row and second column and fourth row and second column.

TABLE I. FIBONACCI SERIES THROUGH RABBIT REPRODUCTION

| | 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | |
|----|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|---|
| 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 4 | 1 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 5 | 1 | 1 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | 1 | 1 | 1 | 0 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 7 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 8 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | |
| 9 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | |
| 10 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| 11 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 12 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

The above concept used to represent the Fibonacci series through integers. The Fibonacci series are zero, one, one, two, three, five, eight, thirteen, twenty-one, thirty-four, fifty-five, eighty-nine, ... [9].

The series derived from the equation 1.

$$F_n = F_{n-1} + F_{n-2} \quad (1)$$

Where $F_0=0$, and $F_1=1$.

The list given in the Table II.

TABLE II. FIBONACCI SERIES

| F ₀ | F ₁ | F ₂ | F ₃ | F ₄ | F ₅ | F ₆ | F ₇ | F ₈ | F ₉ | F ₁₀ | F ₁₁ | F ₁₂ |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----------------|-----------------|-----------------|
| 0 | 1 | 1 | 2 | 3 | 5 | 8 | 13 | 21 | 34 | 55 | 89 | 144 |

The Fibonacci series used to create Fibonacci code. The Fibonacci code involves symbol, code word, and Fibonacci representation. The two initial values 0 and 1 neglected. The symbol 1 represented as F₂. The symbol 2, 3, 4, 5, and 6 represented as F₃, F₄, F₂ + F₄ (1+3), F₅, and F₂ + F₅ (1+5). The remaining list completed using Table II. The code word ends with 11 for all symbol representation. The length of code word for F₂, F₃, and F₄ = two, three, and four. i.e. The length of code word for F_n = n.

TABLE III. CODE WORD FOR SYMBOL [1-34]

| Position→ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Value | Code word= Value + Suffix 1 |
|-----------|---|---|---|---|---|----|----|----|---------|-----------------------------|
| Value→ | 1 | 2 | 3 | 5 | 8 | 13 | 21 | 34 | | |
| Symbol | | | | | | | | | | |
| 1 | 1 | | | | | | | | 1 | 11 |
| 2 | 0 | 1 | | | | | | | 01 | 011 |
| 3 | 0 | 0 | 1 | | | | | | 001 | 0011 |
| 4 | 1 | 0 | 1 | | | | | | 101 | 1011 |
| 5 | 0 | 0 | 0 | 1 | | | | | 0001 | 00011 |
| 6 | 1 | 0 | 0 | 1 | | | | | 1001 | 10011 |
| 7 | 0 | 1 | 0 | 1 | | | | | 0101 | 01011 |
| 8 | 0 | 0 | 0 | 0 | 1 | | | | 00001 | 000011 |
| 9 | 1 | 0 | 0 | 0 | 1 | | | | 10001 | 100011 |
| 10 | 0 | 1 | 0 | 0 | 1 | | | | 01001 | 010011 |
| 11 | 0 | 0 | 1 | 0 | 1 | | | | 00101 | 001011 |
| 12 | 1 | 0 | 1 | 0 | 1 | | | | 10101 | 101011 |
| 13 | 0 | 0 | 0 | 0 | 0 | 1 | | | 000001 | 0000011 |
| 14 | 1 | 0 | 0 | 0 | 0 | 1 | | | 100001 | 1000011 |
| 15 | 0 | 1 | 0 | 0 | 0 | 1 | | | 010001 | 0100011 |
| 16 | 0 | 0 | 1 | 0 | 0 | 1 | | | 001001 | 0010011 |
| 17 | 1 | 0 | 1 | 0 | 0 | 1 | | | 101001 | 1010011 |
| 18 | 0 | 0 | 0 | 1 | 0 | 1 | | | 000101 | 0001011 |
| 19 | 1 | 0 | 0 | 1 | 0 | 1 | | | 100101 | 1001011 |
| 20 | 0 | 1 | 0 | 1 | 0 | 1 | | | 010101 | 0101011 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | | 0000001 | 00000011 |
| 22 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | | 1000001 | 10000011 |
| 23 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | | 0100001 | 01000011 |
| 24 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | | 0010001 | 00100011 |

| | | | | | | | | | | |
|----|---|---|---|---|---|---|---|---|----------|-----------|
| 25 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | | 1010001 | 10100011 |
| 26 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | | 0001001 | 00010011 |
| 27 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | | 1001001 | 10010011 |
| 28 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | | 0101001 | 01010011 |
| 29 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | | 0000101 | 00001011 |
| 30 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | | 1000101 | 10001011 |
| 31 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | | 0100101 | 01001011 |
| 32 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | | 0010101 | 00101011 |
| 33 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | | 1010101 | 10101011 |
| 34 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 00000001 | 000000011 |

A. FEWER AMOUNT OF CHARACTERS

If input message (M) length = 100. Where [M] = [m₁, m₂, ..., m₈] with the occurrence [32, 21, 17, 12, 10, 5, 2, 1]. The probability of each character is as given in the Table IV. The Code Word for each character also provided in the Table V by referring Table III.

TABLE IV. CODE TABLE – CHARACTER OCCURRENCE, PROBABILITY

| CHARACTER | m ₁ | m ₂ | m ₃ | m ₄ | m ₅ | m ₆ | m ₇ | m ₈ |
|-------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| OCCURRENCE | 32 | 21 | 17 | 12 | 10 | 5 | 2 | 1 |
| PROBABILITY | 0.32 | 0.21 | 0.17 | 0.12 | 0.1 | 0.05 | 0.02 | 0.01 |

TABLE V. CODE TABLE – CHARACTER CODE WORD(FEWER AMOUNT)

| Message | m ₁ | m ₂ | m ₃ | m ₄ | m ₅ | m ₆ | m ₇ | m ₈ |
|-------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Probability | 0.32 | 0.21 | 0.17 | 0.12 | 0.1 | 0.05 | 0.02 | 0.01 |
| Code Word | 11 | 011 | 0011 | 1011 | 00011 | 10011 | 01011 | 000011 |

$$\begin{aligned}
 \text{The total number of bits needed} &= 32 * 2 + 21 * 3 + 17 * 4 + 12 * 4 + 10 * 5 + 5 * 5 + 2 * 5 + 1 * 6 \\
 &= 64 + 63 + 68 + 48 + 50 + 25 + 10 + 6 \\
 &= 334 \text{ bits}
 \end{aligned}$$

$$\begin{aligned}
 \text{The size of the input as uncompressed} &= 100 * 8 \\
 &= 800 \text{ bits}
 \end{aligned}$$

B. MORE AMOUNT OF CHARACTERS

INPUT

Dr.Ezhilarasu Umadevi Palani obtained his Post Graduate Degree in Computer Science and Engineering from Anna University, Chennai.

TABLE VI. CODE TABLE – CHARACTER CODE WORD(MORE AMOUNT)

| S.No | Symbol | Occurrence | Probability | Code Word | Code Word Length | Code Word – Total Bits |
|------|---------------------|------------|-------------|-----------|------------------|------------------------|
| 1. | Space character “ ” | 16 | 0.124031 | 11 | 2 | 32 |
| 2. | e | 13 | 0.100775 | 011 | 3 | 39 |

| | | | | | | |
|-----|---|----|----------|----------|---|----|
| 3. | n | 13 | 0.100775 | 0011 | 4 | 52 |
| 4. | i | 12 | 0.093023 | 1011 | 4 | 48 |
| 5. | a | 11 | 0.085271 | 00011 | 5 | 55 |
| 6. | r | 8 | 0.062016 | 10011 | 5 | 40 |
| 7. | t | 5 | 0.03876 | 01011 | 5 | 25 |
| 8. | d | 4 | 0.031008 | 000011 | 6 | 24 |
| 9. | o | 4 | 0.031008 | 100011 | 6 | 24 |
| 10. | s | 4 | 0.031008 | 010011 | 6 | 24 |
| 11. | h | 3 | 0.023256 | 001011 | 6 | 18 |
| 12. | m | 3 | 0.023256 | 101011 | 6 | 18 |
| 13. | u | 3 | 0.023256 | 0000011 | 7 | 21 |
| 14. | g | 3 | 0.023256 | 1000011 | 7 | 21 |
| 15. | U | 2 | 0.015504 | 0100011 | 7 | 14 |
| 16. | E | 2 | 0.015504 | 0010011 | 7 | 14 |
| 17. | C | 2 | 0.015504 | 1010011 | 7 | 14 |
| 18. | . | 2 | 0.015504 | 0001011 | 7 | 14 |
| 19. | l | 2 | 0.015504 | 1001011 | 7 | 14 |
| 20. | v | 2 | 0.015504 | 0101011 | 7 | 14 |
| 21. | c | 2 | 0.015504 | 00000011 | 8 | 16 |
| 22. | D | 2 | 0.015504 | 10000011 | 8 | 16 |
| 23. | P | 2 | 0.015504 | 01000011 | 8 | 16 |
| 24. | b | 1 | 0.007752 | 00100011 | 8 | 8 |
| 25. | G | 1 | 0.007752 | 10100011 | 8 | 8 |
| 26. | S | 1 | 0.007752 | 00010011 | 8 | 8 |
| 27. | z | 1 | 0.007752 | 10010011 | 8 | 8 |
| 28. | p | 1 | 0.007752 | 01010011 | 8 | 8 |
| 29. | f | 1 | 0.007752 | 00001011 | 8 | 8 |
| 30. | y | 1 | 0.007752 | 10001011 | 8 | 8 |
| 31. | , | 1 | 0.007752 | 01001011 | 8 | 8 |
| 32. | A | 1 | 0.007752 | 00101011 | 8 | 8 |

The total number of bits needed

$$= 32 + 39 + 52 + 48 + 55 + 40 + 25 + 24 + 24 + 24 + 18 + 18$$

8

$$+ 21 + 21 + 14 + 14 + 14 + 14 + 14 + 14 + 14 + 16 + 16 + 16 +$$

$$+ 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8$$

$$= 645 \text{ bits}$$

The size of the input as uncompressed

$$= 129 * 8$$

$$= 1032 \text{ bits}$$

ENCODING

The given input “Dr.Ezhilarasu Umadevi Palani obtained his Post Graduate Degree in Computer Science and Engineering from Anna University, Chennai.” after encoding will be

100000111001100010110010011100100110010111011100101100011100110001101001100000111101000111010110001
 1000011011010101110111010000110001110010110001100111011110001100100011010110001110110011011000011
 110010111011010011110100001100011010011010111110100011100110000110000011000110101101111100000
 110111000011100110110111110110011111010011110001110101001100000110101101110011001100000001
 110110110011000000110111100011001100001111001001100111000011101100110110110011100001111000

010111001100011101011100101011001100110001110100011001110110101011011100110100111011010111000101
1010010111101001100101101100110001110110001011

DECODING

Before Decoding

100000110011000101100100111001001100101101100101100011001100011000110000011101000111010110001
10000110110101011101111010000110001110010110001100111011100011001000110101100011011001101100001
1100101110110100111101000011100011010010111101000111001100011000001100011010110111100000
1101110000111001101101111011001111101000111001101011011100011010111001111000100110000001
11011011001100000011011110001100110000111000100110011100011101101101110011101100000001
01011100111000111010111001010110011000111010001100111011010110111001110110000000111000
01010010111101001100101101100110001110110001011

The decoding process read the input, character by character until the number 11 found. Then the entire binary code decoded to corresponding unique character.

Step 1

D1001100010110010011100100101101110010110001110011000110100110000011101000111010110001100001
10110101011101110100001100011100101100011001110111100011001000110101100011011000011110010
1110110100111101000011100011010011010111101000111001100011000001100011010110111100000110111
00001110011011011110110011110100111000111010110100110000011010110111001111000100110000001110110
11001100000011011110001100110000111001001100111000011101100110111001110110001111000010111
0011100011101011110010101100110001110100011001110101011011100110100111011010111000101101001
0111101001100101101100110001110110001011

Step 2

Dr00010110010011100100110010111011100101100011100110001101001100000111010001110101100011000011011
0101011101110100001100011100101100011001110111100011001000110101100011101100110110000111100101110
11010011110100001110001101001101011110100011100110000110000011000110101101111000001101110000
11100110110111101100111101001110001110101001100000110101101110011110001001100000011101101100
110000001101111000110011000011100100110011100001110110011011011100111011000011110000101110011
1000111010111100101011001100011101000111010101101110011010011101101011100010110100101101001
1101001100101101100110001110110001011

Step 3

Dr.0010011100100110010111011100101100011100110001101001100000111010001110101100011000011011010111
10111101000011000111001011000110011101111000110010001101011000110110000111001011101101001
11101000011100011010011010111101000111001100001100000110001101011011110000011011100001110011
01101111011001111010011100011101010100110000011010110111000100110000001110110110001100000
01101111000110011000011100100110011100001110110011011011100111011000011110000101110011100011
0101111001010110011000111010001110110101011011100110100111011010111000101101001011010010111101001
100101101100110001110110001011

Step 4

Dr.E100100110010111011100101100011100110001101001100000111010001110101100011000011011010101110111
010000110001110010110001100111011110001100100011010110001101100001110010111011001011101001111010
0001110001101001101011110100011100110000110000011000110101101111000001101110001110011011011
1110110011111010011100011101010011000001101011011100010011000000111011011001100000011011
110001100110000111001001100111000011101100110111001110000101110011100001011100111000111010111
1001010110011000111010001100111010101101110011010011101101011100010110100101111010011001
1011001100110001110110001011

Step 5

Dr.Ez00101110111001011000111001100011010011000001110100011101011000110000110110101011101110100001
1000111001011000110011101111000110010001101011001101100001110010111011010011110100001110
001101001101011110100011100110001100000110001101011011110000011011100001110011011011110110
011111010011100011101010100110000011010110111000100110000001110110110011000000110111100011
0011000011110010011001110000111011001101110011100001011100111000010111001110001110101111001010

110011001100011110100011001110110101011011100110100111011010111000101101001011111010011001011011001
100110001110110001011

Step 6

Step 7

Dr.Ezhi10010110001110011000110100110000011110100011101011000110000110110101011101111010000110001110
0101100011001110111100011001000110101100011101100110110000111100101110110100111101000011000110100
110101111010001110011000110000011000110101101111000001101110000111001101101111011001111101
0011100011101011010011000001101011011100111100010011000000111011011001100000011011110001100110000
111100100110011100001110110011011011100111001110000111100001011100111000111010111100101011001100
110001110100011001110110101101110011010011101101011100010110100101111010011001011011001100011000
1110110001011

Step 8-129

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$$= 37.5\%$$

Average bits = 645/129

= 5.0 bits per character

IV. RESULT AND DISCUSSION

The compression ratio, space savings and average bits calculated for the fewer amount of data are

$$\begin{aligned}\text{Compression ratio} &= 800/334 \\ &= 200:87 \\ &\approx 2.395:1\end{aligned}$$

$$\begin{aligned}
 \text{Space savings} &= 1 - (334/800) \\
 &= 1 - (87/200) \\
 &= 1 - 0.435 \\
 &= 0.565 \\
 &= 56.5\%
 \end{aligned}$$

$$\begin{aligned}\text{Average bits} &= 334/100 \\ &= 3.34 \text{ bits per character}\end{aligned}$$

The compression ratio, space savings and average bits calculated for the more amounts of data are

Compression ratio = 1032/645
= 344:215
= 1.6:1

$$\begin{aligned}
 \text{Space savings} &= 1 - (645/1032) \\
 &= 1 - (215/344) \\
 &= 1 - 0.625 \\
 &= 0.375
 \end{aligned}$$

V. CONCLUSION

The Fibonacci coding is a data compression technique that based on Fibonacci series. It produces static variable length code for representing the data. The Shannon-Fano coding and Huffman Coding provides dynamic variable length code. Both, the fewer amount and more amount input data produces good compression ratio, space savings, and average bits per character. But for less number of input characters with more probability, Fibonacci coding gives better compression ratio, space savings, and average bits per character.

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